RELAXATION TECHNIQUE AS APPLIED TO ELECTRICAL NETWORK PROBLEM OF "RING DISTRIBUTOR"

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ABSTRACT This paper deals with the relaxational solution of the problem of a "Ring Distributor" In this method a set of linear simultaneous equations in obtained using Kirchoff's laws of networks, and these equations are solved by relaxation method It reveals the usefulness of the relaxational solution of any problem for which a number of linear simultaneous equations can be developed and further it yields a number of required informations simultaneously This is illustrated with an example and the results thus obtained are compared with those calculated by conventional method of network analysis

INTRODUCTION

In low voltage D C. power distribution, the Ring Distributor (Starr, 1946 Cotton, 1948), is used in preference to other systems for having uninterrupted supply with a minimum expenditure on transmission lines or feeders Because in this system any load may be supplied even if there is a break in transmission lines This problem can be solved by the conventional methods used in network analysis But those methods become laborious with the increase of the loading points This paper suggests a method based on relaxation technique (Allen, 1954), and shows the utility of its application for such problems

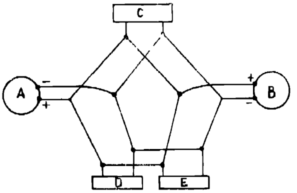

Fig. 1

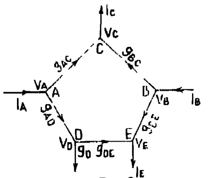

Fig. 2

Here a case of a symmetrical circuit i e each wire of each pair of lines having the same resistance as shown in Fig. 1, is dealt with It can be further simplified by assuming that one wire ring has zero resistance and that each wire in the other ring has twice its actual value of the resistance, so that the equivalent circuit diagram can be drawn as shown in fig 2.

In this method a number linear simultaneous equations are obtained using Kirchoff's laws at each nodal point of Fig 2. These equations are solved very easily applying relaxation method.

PRINCIPLE OF THE METHOD

From the equivalent circuit diagram of the network shown in Fig 2, the following linear simultaneous equations can be written considering the different nodal points viz. A, B, C, D and E.

$$\left.\begin{array}{lll}\text{At} & A, & I_A - (V_A - V_C)g_{AC} - (V_A - V_D)g_{AD} = 0 \\ \text{"} & C, & (V_A - V_C)g_{AC} + (V_B - V_C)g_{BC} - I_C = 0 \\ \text{"} & B, & I_B - (V_B - V_C)g_{BC} - (V_B - V_E)g_{BE} = 0 \\ \text{"} & E, & (V_B - V_E)g_{BE} + (V_D - V_E)g_{DE} - I_E = 0 \\ \text{"} & D, & (V_A - V_D)g_{AD} - (V_D - V_E)g_{DE} - V_D g_D = 0 \end{array}\right\} \qquad (1)$$

Where,

I_A = Current of the generator A

I_B = ... B

I_C = ... load C

I_E = ... E

V_A = Voltage of the generator A

V_B = ... B

V_C = ... load C

V_D = ... D

V_E = ... E

g_{AC} = $1/R_{AC}$, R_{AC} being twice the resistance of each wir[illegible]cting the generator A to the load C

g_{BC} = $1/R_{BC}$, R_{BC} .... B C

g_{BE} = $1/R_{BE}$, R_{BE} .... B .... E

g_{DE} = $1/R_{DE}$, R_{DE} .... load D .... E.

g_{AD} = $1/R_{AD}$, R_{AD} .... generator A .... D.

g_D = $1/R_D$, R_D being the resistance of the load D

The above set of equations after simplification may be written as follows :

$$\left.\begin{aligned}
&I_A - V_A(g_{AC}+g_{AD}) + V_C g_{AC} + V_D g_{AD} = 0 = F_1\\
&V_A g_{AC} - V_C(g_{AC}+g_{BC}) + V_B g_{BC} - I_C = 0 = F_2\\
&I_B - V_B(g_{BC}+g_{BE}) + V_C g_{BC} + V_E g_{BE} = 0 = F_3\\
&V_B g_{BE} - V_E(g_{BE}+g_{DE}) + V_D g_{DE} - I_E = 0 = F_4\\
&V_A g_{AD} - V_D(g_{AD}+g_{DE}+g_D) + V_E g_{DE} = 0 = F_5
\end{aligned}\right\} \quad \ldots \quad (2)$$

where F_1, F_2, F_3, F_4 and F_5 are the residuals, the liquidation of which yields the unknown quantities contained in the relation (2). In liquidating them the required basic unit, block, group operation tables and the relaxation table can be prepared (Allen 1954). In basic unit operation table the changes of the residuals due to unit positive increment to the value of the unknown are written (Table I, operation number 1 to 5). In block operation equal increments to more than one unknown are applied simultaneously (Table I, operation number 6 and 7) and in group operation unequal increments are added simultaneously to the various unknowns (Table I, operation number 8). So the suitable operation blocks or groups to bring about the changes in one or more residuals without affecting the remaining residuals can be easily written and with the help of this operation table (Table I) the relaxation table (Table II) can be obtained to get the residuals liquidated quickly. This is clearly shown with an illustration given below.

ILLUSTRATION

An example worked out by the conventional method (Mem. Staff Dept. Elect. Eng. M I T, 1953) is taken for illustration where $V_A = 230$ volts, $V_B = 220$ volts, $I_C = 1000$ amps, $I_E = 1500$ amps, $R_D = 0.4$ ohm, $R_{AC} = 0.008$ ohm, $R_{BC} = 0.006$ ohm, $R_{BE} = 0.006$ ohm, $R_{DE} = 0.01$ ohm, $R_{AD} = 0.008$ ohm are given. It is required to calculate the line currents, load currents, voltages, resistances and generator currents.

Substituting the numerical values in the relation (2) it can be written as follows

$$\left.\begin{aligned}
&I_A + 125V_C + 125V_D - 57500 = 0 = F_1\\
&-292V_C + 64490 = 0 = F_2\\
&I_B + 167V_C + 167V_E - 73480 = 0 = F_3\\
&100V_D - 267V_E + 35240 = 0 = F_4\\
&-227.5V_D + 100V_E + 28750 = 0 = F_5
\end{aligned}\right\} \quad \ldots \quad (3)$$

In the above illustration the block and group operations are performed after careful study and observation of the unit operation table From those operation tables final relaxation table is obtained The actual procedure followed to obtain the block and group operation or liquidation of the residuals is shown within the bracket | | and the operation number in unit, block, or group operation tables and liquidation step in relaxation tables are shown within the bracket ( ).

After liquidation of the residuals the generator currents and the load voltages are obtained directly from the relaxation table From those and the supplied values of the generator voltages, load currents and the line resistances, the required line currents and load resistances and also the load current can be calculated as shown below

$I_A = 2302.074$ amps

$I_B = 749.736$ "

$V_C = 220.856$ volts

$V_D = 220.727$ "

$V_E = 214.654$ "

$I_{AC} = 1142.875$ amps, where I_{AC} is the current flowing in the line joining the generator A and the load C

$I_{AD} = 1159.875$ " " " I_{AD} .... D

$I_{DE} = 607.290$ " " " I_{DE} .... load D ....E

$I_{BC} = -142.700$ " " " I_{BC} .... generator B ....C.

$I_D = 551.818$ " " " I_D .... of the load D

$R_C = 0.221$ ohm, where R_C is the resistance of the load C

$R_E = 0.143$ " " " R_E ....E.

The above values are seen to be in very good agreement with those obtained by conventional methods of network analysis shown in the table below (Table III)

In the Table III methods I, II and III mean the Relaxation method, conventional method of network analysis and the conventional method followed (with some approximations) in the book (Mem. Staff Dept Elec. Eng., M.I T , 1953) respectively.

TABLE I
Operation Table

Operation number	δI_A	δI_B	δV_C	δV_D	δV_B	δF_1	δF_2	δF_3	δF_4	δF_5
Unit Operation										
(1)	1	—	—	—	—	—	1	0	0	0
(2)	—	1	—	—	—	0	0	1	0	0
(3)	—	—	1	—	—	125	−292	167	0	0
(4)	—	—	—	1	—	125	0	0	100	−227.5
(5)	—	—	—	—	1	0	0	167	−267	100
Block operation										
(6) [(1)+(2)]	1	1	—	—	—	1	0	1	0	0
(7) [(3)+(4)−(5)]	—	—	1	1	1	250	−292	334	−167	−127.5
Group operation										
(8) [2.67 × (4)−(5)]	—	—	—	2.67	1	333.75	0	167	0	507.425

TABLE II
Relaxation

	δI_A	δI_B	δV_C	δV_D	δV_B	F_1	F_2	F_3	F_4	F_5
	I_A =	I_B =	V_C =	V_D =	$V_B = O$	−57500	64490	−73840	35240	28750
(1) [220 × (7)]	—	—	200	220	220	−2500	250	0	−1500	700
(2) [15 × (4)]	—	—	—	15		625	250	0	0	2712.5
(3) [−5.3456 × (8)]	—	—	—	−14.2727	−5.3456	−2409.0940	250	−892.7152	0	0
(4) [0.8562 × (3)]	—	—	0.8562	—	—	−2302.0740	0	−749.7365	0	0
(5) [2302.074 × (1) + 749.7365 × (2)]	2302.0740	749.7365	—	—	—	0	0	0	0	0
	2302.0740	749.7365	220.8562	220.7273	214.6544	0	0	0	0	0

TABLE III

Unknown quantities	Values of the unknown quantities obtained by		
	Method I	Method II	Method III
I_A	2302.074 Amps	2301.600 Amps	2300.000 Amps
I_B	749.736 ,,	747.600 ,,	700.000 ,,
I_{AC}	1142.785 ,,	1143.000 ,,	1100.000 ,,
I_{AD}	1159.875 ,,	1157.785 ,,	1170.000 ,,
I_{DE}	607.290 ,,	607.900 ,,	620.000 ,,
I_{BC}	−142.700 ,,	142.667 ,,	−200.000 ,,
I_{BE}	890.933 ,,	890.330 ,,	880.000 ,,
I_D	551.818 ,,	551.862 ,,	550.000 ,,
V_C	220.856 Volts	220.856 Volts	221.000 Volts
V_D	220.727 ,,	220.737 ,,	221.000 ,,
V_E	211.651 ,,	214.658 ,,	215.000 ,,
R_C	0.221 ohm	0.221 ohm	0.221 ohm
R_D	0.113 ,,	0.113 ,,	0.113 ,,

DISCUSSION

This paper shows the utility of relaxation method in the sense that it yields a number of useful informations (generator voltage, load voltage, etc.) simultaneously. Also this method proves to be of great advantage over the other methods when the number of nodal points increases by the change of network conditions such as increase of the loading points etc. Some differences in the values obtained by this method and that in the book (Mem. Staff. Dept. Elec. Eng. M.I.T.,1953), can be accounted for by the fact that in the conventional method followed in that book the values are rounded off considering the practical aspect of the illustrating problem.

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